

Experimental test for 5th dimension in Kaluza-Klein gravity

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Several electric/magnetic charged solutions (dyons) to 5D Kaluza-Klein gravity on the principal bundle are reviewed. Here we examine the possibility that these solutions can act as quantum virtual wormholes in spacetime foam models. By applying a sufficiently large, external electric and/or magnetic field it may be possible to “inflate” these solutions from a quantum to a classical state. This effect could lead to a possible experimental signal for higher dimensions in multidimensional gravity.

1. Introduction

An important aspect in any gravitational theory is to find experimental tests of the theory. Even General Relativity does not have as wide a range of experimental tests in comparison with solid state theory, for example. In multidimensional (MD) gravity theories this lack of experimental tests is even more acute. Usually, tests of MD gravity theories are connected with effects associated with the presence of the scalar field(s) that occur in this theories. For example, in 5D Kaluza - Klein theory it is possible to show that variations of the 5th coordinate lead to changes in the electrical charge to mass ratio of an elementary particle. This effect is very small since no experiment has found such a change.

In this paper we propose another possible experimental test for MD gravity. The basic idea is :

- The Universe can have MD regions, see Fig.1. This means that ***a piecewise compactification mechanism*** can exist in Nature [1]. Piecewise compactification implies that some parts of the Universe are

regions where one has full MD gravity (5D in our case), while other parts of the Universe are ordinary 4D regions where gravity does not act on the extra dimensions. For this mechanism to be viable it is necessary that at the boundary between these regions a quantum splitting off of the 5th dimension occurs. In regions where gravity propagates in all the dimensions the Universe will appear as a true 5D spacetime. In the regions where gravity does not propagate into the extra dimension one has ordinary 4D spacetime plus the gauge fields of the fibre.

- These MD regions can be inflated from the space-time foam in the presence of very strong external electric+magnetic fields.

2. Wormhole and flux tube solutions in 5D gravity.

In 5D Kaluza - Klein theory there are wormholelike and flux tube solutions [2]. The general form of the metric is:

$$\begin{aligned}
 ds^2 &= e^{2\nu(r)} dt^2 - r_0^2 e^{2\psi(r)-2\nu(r)} \times \\
 &\quad [d\chi + \omega(r)dt + \cos\theta d\varphi]^2 \\
 &\quad - dr^2 - a(r)(d\theta^2 + \sin^2\theta d\varphi^2),
 \end{aligned} \tag{1}$$

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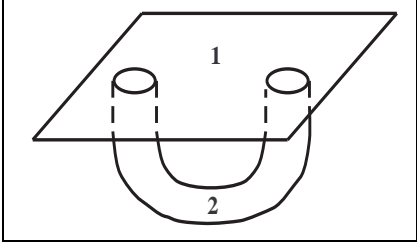


Figure 1. 1 is the 5D spacetime with G_{55} as a non-dynamical variable (in this case 5D gravity is equivalent to the 4D gravity + electromagnetism). 2 is the 5D throat with G_{55} as a dynamical variable.

where χ is the 5th coordinate; $A_t = \omega$ and $A_\phi = \cos \theta$ are the 4D electromagnetic potentials.

A detailed analytical and numerical investigation of this metric gives the following spacetime configurations, whose global structure depends on the relationship between the electric and magnetic fields (see Fig.2):

1. $0 \leq H_{KK} < E_{KK}$. The corresponding solution is **a WH-like object** located between two surfaces at $r = \pm r_0$. The cross-sectional size of this solution (given by $a(r)$) increases as r goes from 0 to $\pm r_0$. The throat between the $r = \pm r_0$ surfaces is filled with electric and/or magnetic flux.
2. $H_{KK} = E_{KK}$. In this case the solution is **an infinite flux tube** filled with constant electrical and magnetic fields, with the charges at $\pm\infty$. The cross-sectional size of this solution is constant ($a = \text{const.}$). In Refs. [3] [2] an exact, analytical form of this solution was found. This solution is almost identical to the 4D Levi-Civita flux tube solution [4] except the strength of the magnetic and electric fields are equal, while in the Levi-Civita solution the two fields can take on any relative value with respect to one another. The restriction that the electric charge equals the magnetic charge is reminiscent of other higher dimensional

soliton solutions [5]. The form of this infinite flux tube configuration also has similarities to the Anti-de Sitter (AdS) “throat region” that one finds by stacking a large number of D3-branes [6].

3. $0 < E_{KK} \leq H_{KK}$. In this case we have **a finite flux tube** between two (+) and (-) magnetic and/or electric charges, which are located at $\pm r_0$. The longitudinal size of this flux tube is finite, but now the cross sectional size decreases as $r \rightarrow r_0$. At $r = \pm r_0$ this solution has real singularities which we interpret as the locations of the magnetic and/or electric charges.

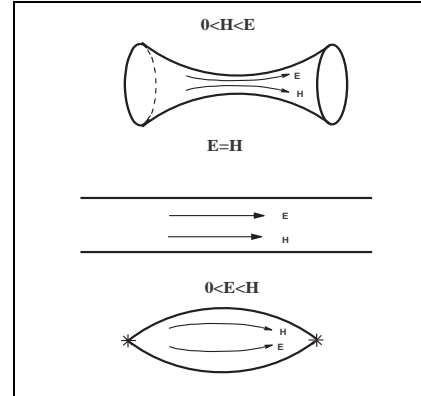


Figure 2. The different types of 5D solutions depends on the relative strengths of the electric and magnetic fields.

3. Creation of dyonic WHs via external electromagnetic fields

The proposed experimental signal for the extra dimensions in MD gravity uses these solutions in the following way:

Composite WHs can appear as a quantum handles (quantum WHs) in the spacetime foam. These quantum structures can be “blown up”

or “inflated” from a quantum state to a classical state by embedding it in parallel E and H fields, see Fig.3

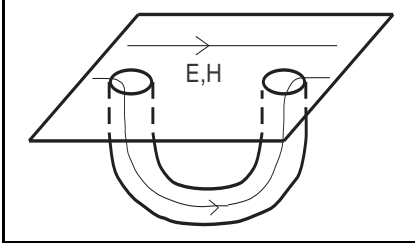


Figure 3. The composite WH filled with the external fields.

These handles are taken as quantum fluctuations in the spacetime foam, which the externally imposed E and H fields can inflate to classical states with some probability.

For an external observer these composite WHs will appear as two oppositely charged dyons, with the charges located on the surfaces where the 4D and 5D spacetimes are matched, see Fig.4. Since

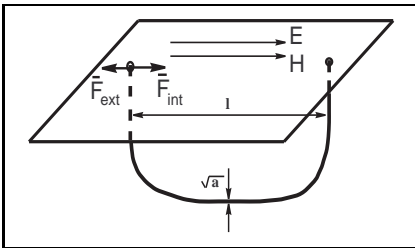


Figure 4. The forces acting on the ends of WH.

one would like these dyonic objects to be well separated, we will consider the case $E \approx H$ ($E > H$). Under these conditions the solution to Einstein's

MD vacuum equations for the metric ansatz is [2]

$$q \approx Q, \quad (2)$$

$$a \approx \frac{q^2}{2} = \text{const}, \quad (3)$$

$$e^\psi \approx e^\nu \approx \cosh\left(\frac{r\sqrt{2}}{q}\right),$$

$$\omega \approx \frac{\sqrt{2}}{r_0} \sinh\left(\frac{r\sqrt{2}}{q}\right) \quad (4)$$

q is the electrical charge and Q the magnetic charge. Both Kaluza - Klein fields are:

$$E \approx H \approx \frac{q}{a} \approx \frac{2}{q} \approx \sqrt{\frac{2}{a}} \quad (5)$$

The cross sectional size of the WH is proportional to q^2 . According to this scenario the external, parallel electric and magnetic fields should fill the virtual WH. In cgs units the fields necessary for forming a composite WH with a cross sectional size a are

$$E \approx H \approx \frac{c^2}{\sqrt{G}} \sqrt{\frac{2}{a}} \quad (6)$$

In order that the charged surfaces of the WH appear as well separated dyons we need to require that the longitudinal distance, l , between these surfaces be much larger than the cross sectional size of the WH, $l \gg a^{1/2}$. Also in order to be able to separate the two ends of the WH as distinct electric/magnetic charged objects one needs the external force to be much larger than the interaction force between the oppositely charged ends. This leads to the following condition

$$F_{ext} = qE + QH \gg \frac{q^2 + Q^2}{l^2} = F_{int}$$

$$\approx 2qE \gg \frac{2q^2}{l^2} \rightarrow l \gg \sqrt{a} \quad (7)$$

If this condition holds than the oppositely charged ends will move apart. Otherwise the ends will come back together and annihilate back into the spacetime foam.

The average value of a for spacetime foam is given by the Planck size $a^{1/2} \approx L_{Pl} \approx 10^{-33}$ cm. Thus the relevant electric field should be $E \approx$

3.1×10^{57} V/cm. This field strength is well beyond experimental capabilities to create. Hence one must consider quantum WHs whose linear size satisfies $a^{1/2} \gg L_{Pl}$. The larger $a^{1/2}$ the smaller the field strength needed. But such large quantum WHs are most likely very rare. If $f(a)$ is the probability density for the distribution for a WH of cross section a then $f(a)da$ gives the probability for the appearance of quantum WH with cross section a . The bigger a the smaller the probability, $f(a)da$. Also the larger the value of the external E and H , the smaller is the cross sectional size a of the WH that can be inflated from the spacetime foam. Thus depending on the unknown probability $f(a)da$ one can set up some spatial region with parallel E and H fields whose magnitudes are as large as technologically feasible, and look for electric/magnetic charged objects whose charges are of similar magnitude.

4. Conclusions

The above estimates seem to give a pessimistic view of “inflating” such quantum WHs via man-made fields. However, certain astronomical objects are able to provide fields which are much stronger than those that can be made in the lab, thus increasing the probability that a quantum WH can be inflated. For example, neutron stars have extremely powerful magnetic fields which are many orders of magnitude larger than can be made in the lab. Also if the neutron star has a companion it can capture plasma from its partner. This captured plasma could generate localized, but strong electric field strengths which in combination with the powerful, global magnetic fields could lead to the “inflation” of the quantum WHs. To an external observer this would appear as the creation of dyonic particles (the end of the flux tubes) with nearly equal electric and magnetic charges. Another possibility is that under the above scenario the magnetic and electric fields of the compact astronomical object (*e.g.* a neutron star) are not large enough for the condition in Eq. (7) to be satisfied. In this case the external fields might start to inflate the quantum WH, but rather than the ends moving apart, they might be pulled back together by the attraction of the op-

positely charged ends. The ends of the quantum WH would thus annihilate back into the vacuum. One can show that just as for QCD flux tubes, that the field energy U of the present flux tubes is proportional to the length of the flux tube, l . Thus, if the maximum separation, l , between the ends of the flux tube were large enough so that one had a substantial field energy, U , then this annihilation could result in an intense burst of gamma ray photons. Such a mechanism could be linked with the gamma ray burster phenomenon, which is often associated with compact astronomical objects such as inspiraling, binary neutron stars or naked singularities [7].

It is also possible that such handles were formed in the very early Universe and then were inflated during the inflationary phase. In order for these handles not to have evaporated via Hawking radiation they must be sustained by being embedded in powerful external magnetic/electric fields created, for example, by binary neutron stars.

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